Application of Geostatistics to Mineral Resource Modeling and Estimation: Case Study of Gofolo Hill Iron Ore Deposit, Western Liberia

Application de la Géostatistique à la Modélisation et à l'Estimation des Ressources Minérales: Étude de Cas du Gisement de Minerai de Fer de Gofolo Hill, dans l'Ouest du Libéria

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ABSTRACT: Geostatistical methods are essential in accurate mineral resource estimation, as they account for spatial correlations and uncertainties. This study evaluates the mineral resource of the Gofolo Hill deposit in Western Liberia using the ordinary kriging method, and in addition, applies two traditional estimation methods, the inverse distance weighting method and the nearest neighbor polygon method, for comparative analysis. The study uses 39 reverse circulation drill holes with 200m x 60m grid spacing. The ordinary kriging method estimated a resource of 17.169 million tonnes with an average grade of 35.90%. In comparison, the inverse distance weighting method estimated 16.975 million tonnes at an average grade of 35.53%, while the nearest neighbor polygon method estimated 14.757 million tonnes at an average grade of 38.55%. The results show that the ordinary kriging method provides the most accurate estimates, followed by the inverse distance weighting method, with the nearest neighbor polygon method showing the least precision. The findings emphasize the importance of a geostatistical approach in resource estimation and support their application in mineral evaluation.

KEY WORDS: Iron ore, Gofolo Hill, Resource estimation, Ordinary Kriging, Inverse Distance Weighting, Nearest neighbor polygon.

RÉSUMÉ: Les méthodes géostatistiques sont essentielles pour une estimation précise des ressources minérales, car elles prennent en compte les corrélations spatiales et les incertitudes. Cette étude évalue les ressources minérales du gisement de Gofolo Hill, situé dans l'ouest du Liberia, en utilisant la méthode du krigeage ordinaire, tout en appliquant deux méthodes d'estimation traditionnelles – la méthode de pondération inverse de la distance et la méthode des polygones du plus proche voisin – à des fins de comparaison. L'étude s'appuie sur 39 forages à circulation inverse, espacés selon une maille de 200 m x 60 m. La méthode du krigeage ordinaire a permis d'estimer une ressource de 17,169 millions de tonnes avec une teneur moyenne de 35,90 %. En comparaison, la méthode de pondération inverse de la distance a estimé 16,975 millions de tonnes avec une teneur moyenne de 35,53 %, tandis que la méthode des polygones du plus proche voisin a estimé 14,757 millions de tonnes avec une teneur moyenne de 38,55 %. Les résultats montrent que le krigeage ordinaire fournit les estimations les plus précises, suivi par la méthode de pondération inverse de la distance, la méthode des polygones montrant la précision la plus faible. Ces résultats soulignent l'importance d'une approche géostatistique dans l'estimation des ressources et confirment son utilité dans l'évaluation minière.

MOTS CLÉS: Minerai de fer, Gofolo Hill, Estimation des ressources, Krigeage ordinaire, Pondération inverse de la distance, Polygone du plus proche voisin.

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1. Introduction

Historically, with four operational iron ore mines in the 1960s producing 15 million tonnes of iron ore, Liberia became Africa's top iron ore producer and one of the world's leading iron ore exporters (Swindell, 1965, 1967). By 1980, production increased to 20 million tonnes of iron ore (Wright, 1986). Thereafter, production significantly declined up to the start of the Liberian civil war in 1989, which led to the closure of major mining operations (Gunn et al., 2018). The civil war lasted for fourteen years and ended in 2003. A transitional political period from 2003 to 2005 set the pace for establishing a democratically elected government in 2006. Since the end of the war, Arcelor Mittal has been the sole producer of iron ore in Liberia, commencing production in 2011 and increasing output to approximately 5 million tonnes by 2021, ranking Liberia the world's 24th largest iron ore producer (Idoine et al., 2024). This research is motivated by the need to explore multiple iron ore deposits in Liberia, evaluate their resources to inform feasibility studies, and advance projects from exploration to exploitation, thereby enhancing Liberia's iron ore production and economic development.

Mineral resource estimation plays a critical role in progressing from mineral exploration to commodity production (Jowitt & McNulty, 2021). Mineral estimation, made possible through the interpretation of quality geological data and economic considerations, determines the viability of a mineral deposit. Precision and accuracy are paramount in mineral resource estimation to avoid unrealistic financial expectations, as these estimations underpin mining operations (Abuntori et al., 2021; Jafrasteh et al., 2018; Jones et al., 2018; Truong et al., 2019).

Mineral resource estimation has evolved significantly since its inception in the early 1900s. Initially, simpler techniques such as the classic polygonal method were employed. Over time, more advanced methods like inverse distance weighting (IDW) and more robust geostatistical kriging were developed, bringing greater precision and reliability to resource estimation.

The traditional methods - nearest neighbor polygon (NNP) and inverse distance weighting (IDW) - also referred to as deterministic methods are widely used during the early stage of mineral resource estimation because of their simplicity and speed. These methods assume that nearby points have a stronger influence on interpolated values than distant points (Eldeiry et al., 2011; Webster & Oliver, 2007). The inverse distance weighting (IDW) method uses distance-weighted averages to estimate unknown values, while the nearest neighbor polygon (NNP) method assigns the value of the closest sampled point to the unknown point. However, these methods fail to account for spatial relationships among data points and cannot quantify estimation uncertainty, often leading to subjective resource assessments (Muktibodh, 2014).

Geostatistical methods address these limitations by incorporating spatial relationships and quantifying estimation uncertainty (Ali Akbar, 2012; Rossi & Deutsch, 2014). The geostatistical kriging method uses a semi-variogram to model spatial relationships, enabling more accurate and unbiased resource estimates (Boroh et al., 2022; Coletti et al., 2022; De Carvalho & Da Costa, 2021; Guo et al., 2022; Kumar et al., 2023). By minimizing error variance, kriging also provides estimates with a measure of prediction uncertainty. Geostatistical estimation is based on the theory of Regionalized Variables (variables with geographical location, spatial position, and correlation), which Prof. Georges Matheron, (1963, 1965, 1971) developed based on the practical work carried out by Krige (1951) in determining the ore grades from drill cores in a South African gold mine.

The geostatistical ordinary kriging (OK) method, inverse distance weighting (IDW) method, and nearest neighbor polygon (NNP) method are grade interpolation methods widely used for mineral resource estimation, validation studies, and comparative analysis (Afeni et al., 2021; Bargawa &

Tobing, 2020; De-Vitry, 2003; Gong et al., 2014; Kasmaee et al., 2010; Mallick & Choudhary, 2019; Shahbeik et al., 2014). Research comparing these methods has yielded mixed results. In some studies, the IDW is superior to ordinary kriging geostatistical methods (Eldeiry et al., 2011; Moghaddam et al., 2018; Nalder & Wein, 1998; Spokas et al., 2003; Weber & Englund, 1992), while others suggest that geostatistical ordinary kriging provide superior estimation (Abed et al., 2014; Buchanan & Triantafilis, 2009; Kimleang et al., 2017; Mallick & Choudhary, 2019; Milillo & Gardella, 2008; Shahbeik et al., 2014; Yasrebi et al., 2009).

This study focuses on three main objectives. First, it aims to model the orebody implicitly. Second, it evaluates the Gofolo Hill iron ore deposit in Western Liberia using the geostatistical ordinary kriging method. Third, it compares the accuracy of the OK method with the IDW and nearest neighbor polygon methods, providing valuable insights into the strengths and limitations of these approaches for mineral resource estimation.

2. Study Area

2.1 Location

The study area, Gofolo Hill, is located in Western Liberia, Grand Cape Mount County, between latitudes 6°52’46” N and 6°53'11” N and longitudes 11°14’19” W and 11°13’25” W (Figure 1). It lies approximately 80km northwest of Liberia's capital, Monrovia, and 20km from the nearest coastline (Klah-Wilson, 2023).

The Gofolo Hill, along with two other deposits (Zaway and Koehnko), is aligned along a mineralized strike that includes the historic Bomi Hills iron ore mine (about 25 km) and Bong Range iron ore mine (about 80 km) (Figure 2) (Klah-Wilson, 2023).

2.2. Geological setting

The Gofolo Hill deposit is situated along the regional northwest trending Todi Shear zone on the edge of the West African craton (Figure 2). The West African craton comprises Precambrian-aged rocks – the primary geological setting of iron ore deposits worldwide. The Todi shear zone forms the boundary between the Liberian age province (2.8 – 3.3 Ga) of Archean age rocks in the north and the Pan African age province (500 Ma) of younger rocks along the Liberian coast (Gunn et al., 2018; Klah-Wilson, 2023).

Iron mineralization in the area is attributed to itabirite, an indication of banded iron formation (BIF) that has undergone regional metamorphism (folding and faulting) and recrystallization. The mineralization has been shaped due to three major tectonothermal deformation events (Gunn et al., 2018; Hadden, 2006; Klah-Wilson et al., 2023; Kromah, 1974) -

1. Liberian orogeny (2.8 – 3.3 Ga) - responsible for the initial infolding and recrystallization of Banded Iron formation into itabirite,
2. Eburnean orogeny (2.1 – 2.2 Ga) - responsible for compression and refolding of itabirite, and
3. Pan-African orogeny (500 Ma) - responsible for further recrystallization and coarsening of itabirites

The iron ore mineralization at Gofolo Hill consists mainly of hematite and magnetite within coarse-grained itabirite, indicative of high-grade potential (Gunn et al., 2018). The itabirite is of both oxide and silicate type and is likely of Archean or Paleoproterozoic origin. Additionally disseminated magnetite with sedimentary sequence (interbedded metasediments) also forms part of the deposit. The Gofolo Hill deposit is underlain by the granitic gneiss basement of the stable West African Craton.

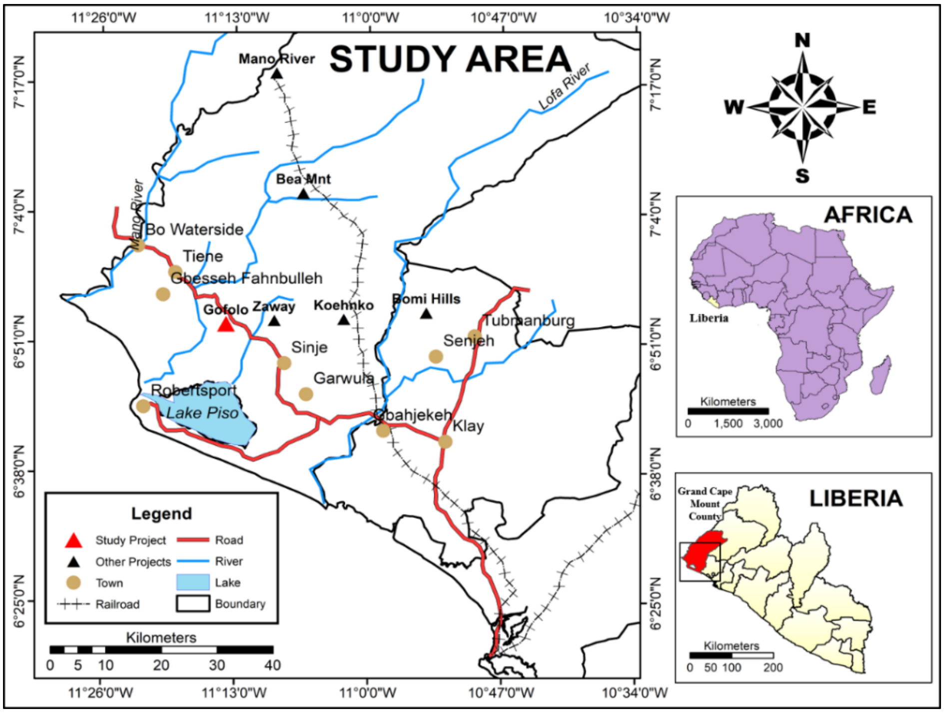

Figure 1 Location map of Gofolo Hill Iron Ore Deposit in Grand Cape Mount County, Western Liberia.

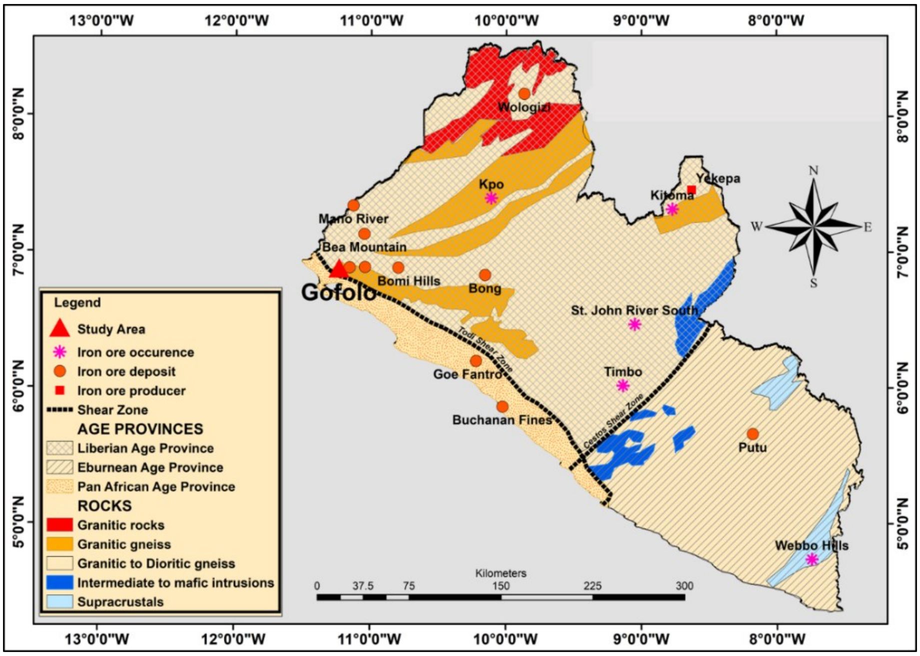

Figure 2 Regional geological map of Liberia showing rock types, age provinces, and notable iron ore deposits and occurrences including the Gofolo Hill iron ore deposit in Western Liberia.

3. Samples and Methods

3.1. Samples and Database

Thirty-nine (39) reverse circulation drill holes, spaced at 200 m x 60 m were used in this study for mineral resource modeling and estimation (Figure 3). The drill holes have depths ranging from a minimum of 42 m to a maximum of 156 m, with an average depth of 87.9 m. Thirty-six (36) of the holes were drilled at a 50-degree dip, while three were drilled vertically (90-degree dip). All holes were surveyed using the UTM WGS_29N coordinate system. Only the iron (Fe) content concentration in the samples' geochemical analysis was used in this study.

The geological database, comprising of relatable tables containing drillhole records, was created, validated, and composited at 2m intervals. The database includes four primary tables: collar, survey, assay, and geology. The collar table contains location data, the survey table includes orientation details, the assay table records the iron (%Fe) elemental analysis, and the geology table contains the drillhole rock types.

3.2. Data Processing Software

Microsoft Excel: used for pre-processing the collar, survey, assay, and geology tables to ensure data accuracy and consistency.

ESRI ArcGIS 10.8.2: utilized to produce a location map and a regional geological map of the study area and Liberia.

Datamine Studio RM 1.13: employed for data analysis, 3D modeling, and resource estimation.

3.3 Data Preprocessing

Effective resource estimation depends heavily on the quality and reliability of data from drilled samples. Prior to applying geostatistical estimation methods (e.g. ordinary kriging) or deterministic methods (e.g. inverse distance weighting and nearest neighbor polygon), it is important to thoroughly preprocess data to remove errors, ensure data consistency, and enhance statistical validity (Dash et al., 2023; Vinutha et al., 2018).

The raw datasets of the Gofolo Hill iron ore deposit, comprising of collar, survey, geology, and assay tables were inspected for missing values, duplicates, and typographical errors. Outlier detection was conducted using multiple descriptive statistics such as mean, standard deviation, skewness, kurtosis, and the interquartile range (IQR) method. The interquartile range (IQR) is calculated using Equation (1):

$$IQR = Q3 - Q1 \qquad (1)$$

Where:

IQR is the interquartile range, Q1 is the first quartile (25th percentile), and Q3 is the third quartile or 75th percentile.

The lower or upper fences are determined by the Equation (2) and Equation (3):

$$Lower\,fence = Q1 - 1.5 * IQR \qquad (2)$$

$$Upper\,fence = Q3 + 1.5 * IQR \qquad (3)$$

Values below the IQR lower bound (lower fence) or above the IQR upper bound (upper fence) are considered outliers.

Although deterministic methods such as inverse distance weighting (IDW) and nearest neighbor polygon (NNP) do not necessarily require data normality, geostatistical methods like ordinary kriging require that the input data are normally distributed. Normality assessment of the data was made through analysis of raw data statistics (coefficient of variation, standard deviation, skewness, kurtosis, mean, median) as well as histogram.

3.4. Resource estimation or interpolation methods

The geostatistical ordinary kriging (OK) method, and two non-geostatistical methods, the inverse distance weighting (IDW) method, and the nearest neighbor polygon (NNP) method, were used to estimate the iron ore deposit of the Gofolo Hill. Ordinary kriging estimation is generally based on a variogram analysis to quantify spatial relationships and make better estimates (Boroh et al., 2022; Coletti et al., 2022; De Carvalho & Da Costa, 2021; Guo et al., 2022; Kumar et al., 2023). The ordinary kriging spatial interpolation method is noted for providing minimum error variance (Yamamoto, 2005). The IDW and NNP are two deterministic methods that predict unknown values based on closeness or distance to known points.

3.4.1. Geostatistics

Geostatistics works best with spatially correlated samples to classify the deposit's natural characteristics and mineralization trend. The most commonly used geostatistical method is the ordinary kriging method (Lefohn et al., 1988), which is selected for this study. Ordinary kriging interpolation utilizes a semi-variogram to generate the best linear unbiased estimate (BLUE) at each location (Ali Akbar, 2012; Boroh et al., 2022; Klah-Wilson et al., 2023; Mallick & Choudhary, 2019; Negreiros et al., 2010). A semi-variogram is configured by comparing one sample value to all others at constantly increasing intervals or lags. The semi-variogram is computed using Equation (4) (Abuntori et al., 2021).

$$\gamma(h) = \frac{1}{2}\sum_{i=1}^{n}[Z(X_i) - Z(X_i + h)]^2 \qquad (4)$$

Where:

y(*h*) is the semivariogram, $Z(X_i)$ is the grade at a point (X_i) in space, $Z(X_i+h)$ is the grade at another location (lag distance), and n is the pairing number.

Once the semi-variogram is modeled, grade interpolation and estimation are done using the geostatistical ordinary kriging method.

The general equation for ordinary kriging estimation is calculated using Equation (5) (Boroh et al., 2022)

$$Z^* = \sum_{i=1}^{n} \lambda_i Z(x_i) \qquad (5)$$

Where:

Z^* is the estimated value, λ is the sample weight coefficient, and Z represents the individual values at sample points.

The weights must equal 1 to fulfill the unbiased situation (Boroh et al., 2022).

$$\sum_{i=1}^{n} \lambda_i = 1 \tag{6}$$

The kriging weights are obtained by the system of equations that relates the semi-variogram values between sample points:

$$\begin{bmatrix} \gamma(x_1, x_1) & \gamma(x_1, x_2) & \cdots & \gamma(x_1, x_n) & 1 \\ \gamma(x_2, x_1) & \gamma(x_2, x_2) & \cdots & \gamma(x_2, x_1) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma(x_n, x_1) & \gamma(x_n, x_2) & \cdots & \gamma(x_n, x_n) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \\ \mu \end{bmatrix} = \begin{bmatrix} \gamma(x_1, x_0) \\ \gamma(x_2, x_0) \\ \vdots \\ \gamma(x_n, x_1) \\ 1 \end{bmatrix} \tag{7}$$

3.4.2. Inverse Distance Weighting

The inverse distance weighting (IDW) method can evaluate deposits with a wide range of grade variability, from low to high-grade variability. It applies a weighting factor based on an inverse distance function of each sample with a set of known sample values about the central point of an unknown ore block (Klah-Wilson, 2023; Rahman et al., 2010). A general assumption is grounded in the First Law of Geography, which states that a sample value would decrease as one moves away from a point and increase as one moves toward that point (Eldeiry et al., 2011; Webster & Oliver, 2007). The IDW method is a function of distance but uses the inverse of the distance to interpolate unknown points, hence the name, inverse distance weighting method. Typically, the power functions of 2 and 3 are commonly used, with this study utilizing a power of 2. The general mathematical formula for the IDW method is given in Equation (8) (Rossi & Deutsch, 2014):

$$Z^* = \sum_{i=1}^{n} \frac{Z_i}{d_i^n} \Bigg/ \sum_{i=1}^{n} \frac{1}{d_i^n} \tag{8}$$

Where:

Z^* is the estimated value, Z_i is the value of the sample at location i; d_i is the separation distance from point i to the point of reference, and n is the power index (a positive integer).

The value of "n" is chosen arbitrarily but is often based on the type of deposit being dealt with.

3.4.3. Nearest Neighbor Polygon

The nearest neighbor polygon (NNP) method is amongst the simplest and straightforward interpolation techniques for calculating unsampled locations. It is the most common computerized polygonal method. The nearest neighbor polygon method operates by predicting the attributes of unsampled points or blocks by assigning values directly based on grade from a nearby point or block. This means only one point or the nearest sampled point value is assigned to the point that is being estimated. The NNP method is mainly useful in situations where other robust interpolation methods are not effectively applicable in predicting or estimating the outcome of data (Klah-Wilson, 2023). Additionally, it can be used as a complementary method to show that the robust interpolation methods of estimation are within an accepted range, thus proofing their precision and accuracy. The mathematical representation of the NNP method is provided in Equation (9).

$$Z^* = Z_{nearest} \tag{9}$$

Where:
Z* is the estimated value at the unsampled location, and
$Z_{nearest}$ is the grade at the nearest sample point

3.5. Cross Validation

To validate the ordinary kriging interpolation method, the slope of regression histogram was used to compare estimated grade values with the actual grade value. The goal is to have a mean slope or value of 1 or nearly 1. Conditional bias occurs if the mean slope value is less than 0.5 (underestimation) or greater than 1.5 (overestimation). Cross validation of the three methods was performed using the global mean grade difference values, coefficient of variation, and calculated standard error. The standard error is given by the Equation (10).

$$SE = \frac{\sigma}{\sqrt{n}} \tag{10}$$

Where SE is the standard error; σ is the sample standard deviation, and n is the number of samples.

4. Results and Discussion

4.1. Statistical Analysis

Statistical analysis of 39 drill holes is shown in Table 1, with their location shown in Figure 3. The dataset consists of 1525 samples. The minimum and maximum Iron (Fe) grade are 0.94% and 60.32%, respectively. The mean grade is 20.46%, while the median is 19.21%. The interquartile range (Q3 – Q1 or 75th percentile – 25th percentile) is 20.90. Based on the interquartile method, the lower bound and upper bound were calculated as -22.05 and 61.53, respectively, suggesting that the dataset contains no extreme values or outliers.

The coefficient of variation (CV) and standard deviation are 0.62 and 12.77, respectively, indicating minor variability in the dataset. A CV value below 1.5 supports the assumption of a single domain for modeling and estimation while a CV above 1.5 would suggest the need for multiple domains to achieve accurate resource estimates. The skewness value of 0.49 indicates a mildly right skewed data, with a slightly longer tail on the right-hand side of the distribution. Skewness values between -0.5 and +5 are generally considered approximately symmetrical. The kurtosis value of -0.47 suggests a platykurtic distribution, meaning the data has lighter tails and flatter peak in comparison to a perfectly normal distribution. A kurtosis value close to zero (-0.47 in our case) and not highly negative or positive suggests approximate normality. Together, the low coefficient of variation (CV) value, the skewness and kurtosis close to zero, the mean and median with small difference, and the histogram (Figure 4) further prove that the data distribution approximates a normal distribution. Summary statistics are shown in Table 1.

4.2. Variography

A variogram is needed for geostatistical ordinary kriging estimation to show the spatial relationship or variability among data (Boroh et al., 2022; Coletti et al., 2022). It also shows whether the deposit is isotropic or anisotropic. Experimental variograms were computed for Fe and fitted against a spherical theoretical model. The spherical model is the most common variogram model in geostatistics. The variogram analysis revealed that the Gofolo Hill deposit exhibits anisotropy because the range values differ in different directions. A mild anisotropy is shown in the horizontal direction (X vs Y) while a clearer anisotropy is shown between horizontal and vertical directions (X/Y vs Z). Parameters of the semi-variogram fitted using a spherical model are shown in Table 2.

The variogram models (Figure 5) with azimuth 0° (north-south direction) and azimuth 90° (east-west direction) were applied for grade interpolation.

Table 1 Summary statistics of raw data.

Parameter	Value
Total Samples	1525
Minimum	0.94
Maximum	60.32
Mean	20.46
Median	19.21
Coefficient of Variation	0.62
Standard Deviation	12.77
Skewness	0.49
Kurtosis	-0.47
25th Percentile	9.28
50th Percentile	19.21
75th Percentile	30.18

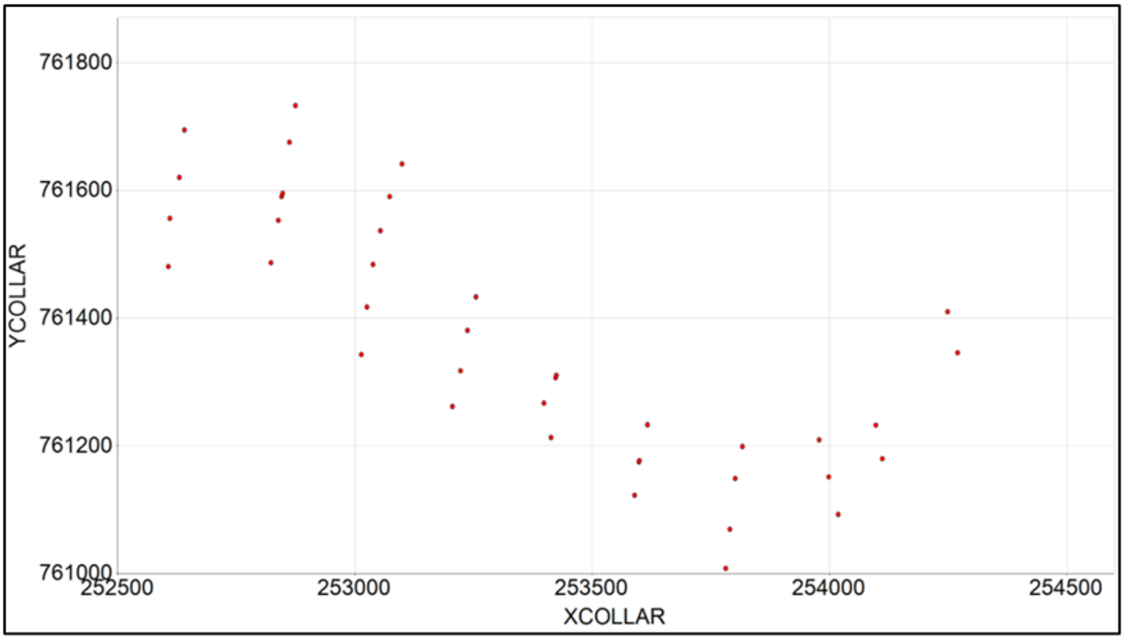

Figure 3 Spatial distribution of drillholes across the Gofolo Hill iron ore deposit showing collar positions used for geological modeling and resource estimation.

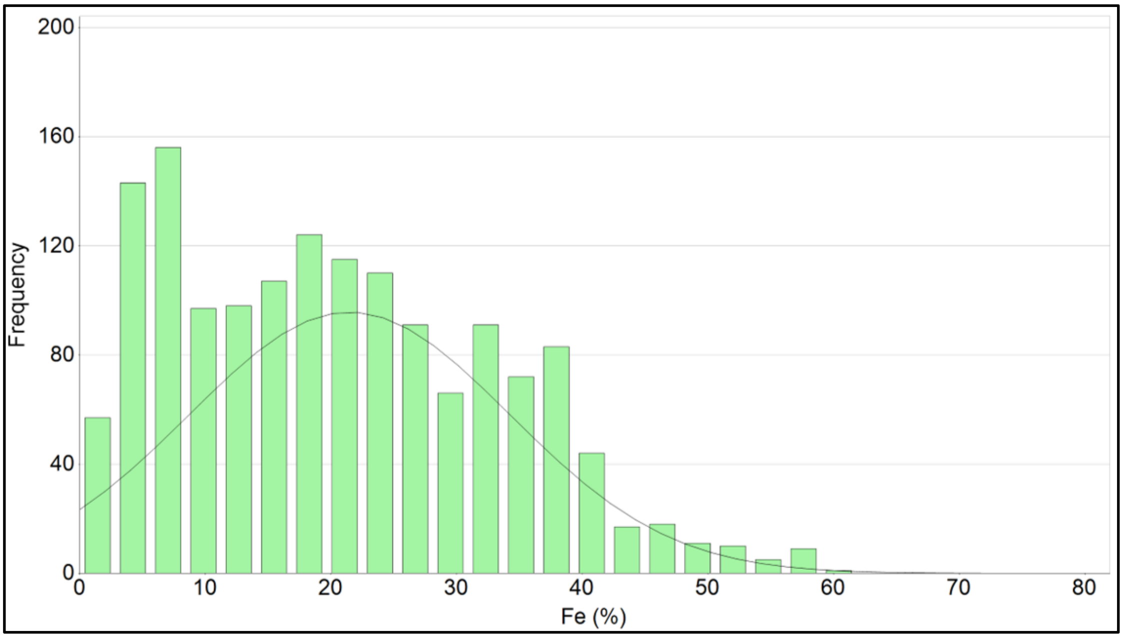

Figure 4 Histogram showing the frequency distribution of iron (Fe) grades from drill hole samples at Gofolo Hill.

Table 2 Parameters of a single structure spherical variogram model showing anisotropy.

Type	Nugget	Sill	Range		
			(X)	(Y)	(Z)
Spherical Model	35.82	126.72	83.50	71.40	58.40

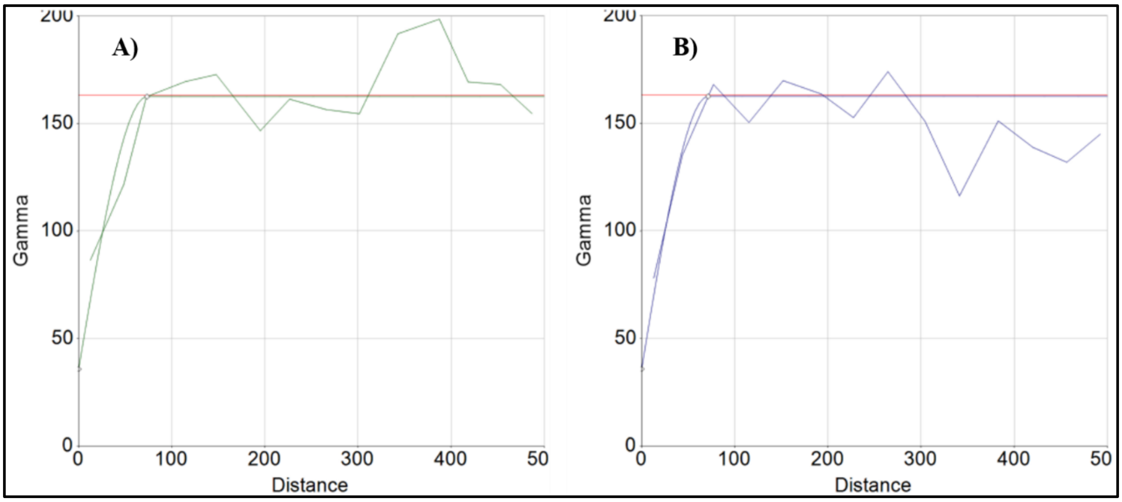

Figure 5 Illustration of experimental variogram fitted with theroretical variogram. A) variogram model with azimuth 0° or north-south direction B) variogram model with azimuth 90° or east-west direction.

4.3. Geological Model and Block Model

A 3D geological model was constructed to visualize the extent and shape of the orebody below the surface. To differentiate ore from waste, a 30% Fe grade cut-off was applied. The ore was modeled carefully after the 30% grade cut-off threshold. Referencing a drill hole spacing of 200m x 60 m, a block size of 50m x 20m x 5m (Figure 6)was chosen for block model and estimation, aligning with David's (1977) guideline that block size should range from ¼ to ½ of the drill hole spacing.

An implicit 3D geological modeling approach was preferred over the explicit approach due to its dynamic ease of update and efficiency. Unlike the explicit method, which requires significant time for manual cross-section compilation, the implicit method allows updates and adjustments. The geographical extent and cell size of the block model are presented in Table 3.

To validate the block model precision and accuracy, the volume of the geological model was compared to that of the block model. The percent difference was 0.29% (Table 3), which is well below the 2% maximum threshold, confirming the accuracy and reliability of the 3D geological and block models.

Table 3 Block model properties showing geographical extent, cell size, and validation details.

Model size					Validation of geological model and block model			
	Cell size (m)	**Min (m)**	**Max (m)**	**Cell count**	**Geological model Volume (m^3)**	**Block model volume (m^3)**	**Volume Diff (m^3)**	**(%) Diff**
X	50	252506.5	254156.5	33	6,248,455.49	6,230,099.17	18,356.31	0.29
Y	20	760908.10	761888.10	49				
Z	5	-135.42	179.58	63				
				101,871				

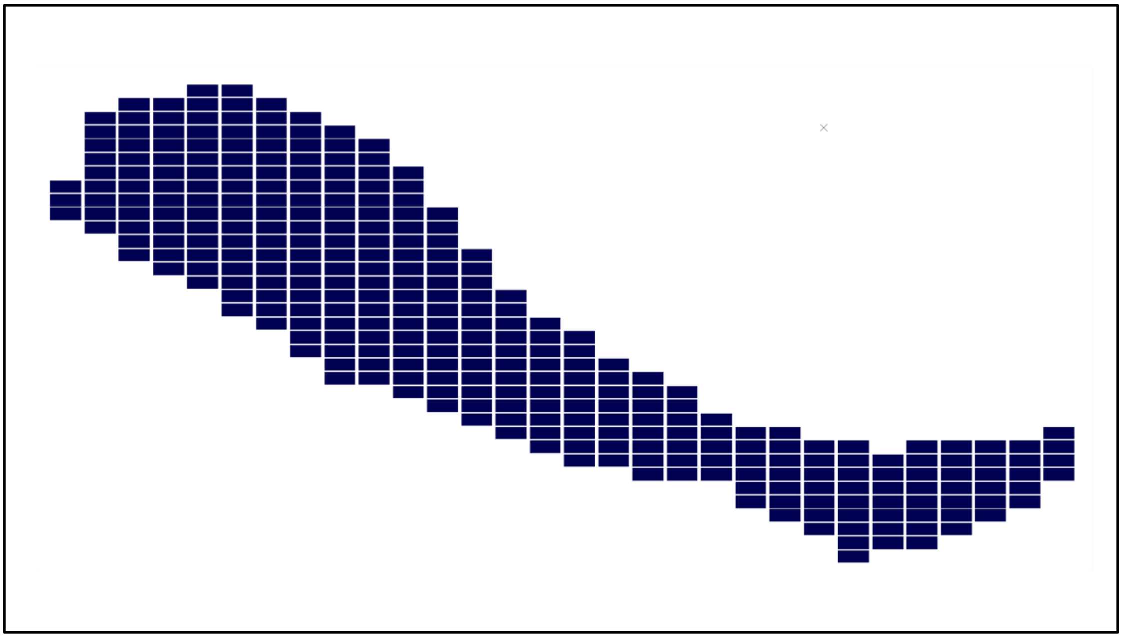

Figure 6 A plan view of the block model generated from 3D geological model of Gofolo Hill deposit.

4.4. Estimation of Resource

Three interpolation methods-geostatistical ordinary kriging (OK), inverse distance weighting (IDW), and nearest neighbor polygon (NNP)-were employed to estimate the grade, volume, and tonnage of the Gofolo Hill Iron ore deposit. The resource estimation includes calculating the ore's grade, volume, and tonnage. The resource estimation utilized a global cut-off grade of 30% Fe and a bulk density of 3.0 kg/m^3 to interpolate the ore blocks. Both the cut-off grade and bulk density are critical factors in resource estimation: the cut-off grade represents the minimum ore grade at which mining and processing are economically viable (Rendu, 2014), while bulk density, defined as the ratio of mass to volume, is essential for calculating tonnage (Clout & Manuel, 2015).

The ordinary kriging (OK) method, which incorporates spatial relationships and trends in sample data through semi-variogram parameters, estimated the deposit at 17.169 million tonnes with an average Fe grade of 35.90%. The inverse distance weighting (IDW) method produced a comparable estimate of 16.975 million tonnes at an average grade of 35.53% Fe. In contrast, the nearest neighbor polygon (NNP) method overestimated the Fe grade at 38.53% while underestimating the tonnage at 14.757 million tonnes. This discrepancy highlights the tendency of the nearest neighbor polygon (NNP) method to overestimate grade due to lack of smoothing and spatial weighting. The results of the grade, volume, and tonnage estimation for all three methods are summarized in Table 4.

Table 4 Resource estimation and summary statistics of the OK, IDW, and NNP methods.

	Resource Estimation			Statistics						
	Grade (%)	Volume (m^3)	Tonnage (tonnes)	Min	Max	Range	Mean	Median	CV	SD
OK	35.9	5,723,145	17,169,435	16.91	51.71	34.80	35.00	34.48	0.12	4.32
IDW	35.53	5,658,375	16,975,126	16.99	54.91	37.92	34.54	34.04	0.13	4.36
NNP	38.53	4,919,024	14,757,072	8.34	60.32	51.98	35.17	34.93	0.24	8.46

4.5. Comparative Study

The three estimation techniques - ordinary kriging (OK) geostatistical method, inverse distance weighting (IDW) method, and the nearest neighbor polygon (NNP) method - are compared using, grade-volume-tonnage results, summary statistics, visualization of grade blocks, and correlation coefficients.

4.5.1. Comparison of grade, volume, and tonnage results

The grade-volume-tonnage results of the ordinary kriging, inverse distance weighting, and nearest neighbor polygon methods are summarized in Table 4.

The ordinary kriging method estimates of the grade, volume, and tonnage are 35.9%, 5,723,145 m^3, and 17,169,435 tonnes, respectively. The inverse distance weighting method estimates of the grade, volume, and tonnage are 35.53%, 5,658,375 m^3, and 16,975,126 tonnes, respectively. The nearest neighbor polygon method estimates of the grade, volume, and tonnage are 38.53%, 4.919,024 m^3, and 14,757,072 tonnes, respectively. The difference in tonnage between the ordinary kriging and the inverse distance weighting method is 0.19 million tonnes, whereas the difference between the ordinary kriging method and the nearest neighbor polygon method is 2.41 million tonnes. Generally, the grade, volume, and tonnage results of the ordinary kriging method are more closely comparable to the results of the inverse distance weighting method than those of the nearest neighbor polygon method. This indicates greater similarity in results between OK and IDW, reflecting their comparable interpolation approaches.

4.5.2 Comparison of summary statistics

The summary statistics of the ordinary kriging (OK), inverse distance weighting (IDW), and nearest neighbor polygon (NNP) were compared using key parameters: minimum grade, maximum grade, range, mean, median, coefficient of variation (CV), and standard deviation (SD).

OK and IDW reported comparable minimum grades of 16.91% and 16.99%, respectively, while NNP recorded a significantly lower minimum grade of 8.34%. The maximum grade values were 51.71% for OK, 54.91% for IDW, and a notably higher 60.32% for NNP. The range values were 34.79% (OK), 37.93% (IDW), and 51.98% (NNP), with the broader range for NNP reflecting its extreme minimum and maximum values. The OK and NNP methods showed a more comparable mean of 35.00% and 35.17%, respectively. The IDW mean grade is slightly lower, 34.54%. A median (50th percentile) of 34.48 for the OK method, 34.04 for the IDW method, and 34.93 for the NNP method. A more comparable standard deviation of 4.32 for the OK method and 4.36 for the IDW method were reported, whereas the NNP method shows a higher standard deviation of 8.46. Similarly, the coefficient of variation (CV) values are more comparable for the OK method (0.12) and the IDW method (0.13), while the NNP has a considerably higher CV is 0.24, indicating greater variability.

Although the NNP method showed a similar mean to the OK method, the IDW method performed better and showed more similarity to the OK method in other statistical parameters such as minimum grade, maximum grade, range, median, SD, and CV. This suggests that IDW provides results more comparable to OK, while NNP introduces greater variability and extremes in the grade distribution. The full summary statistics for these methods are detailed in Table 4.

4.5.3 Visualization of grade blocks

A visual comparison is made of the estimated grade blocks by the ordinary kriging, inverse distance weighting, and nearest neighbor polygon methods (Figure 7). Grade blocks are categorized by color: red represents grades above 30% Fe, yellow for grades between 25% to 30% Fe, green for 20% to 25% Fe, and blue for grades below 20% Fe. The blocks of the three methods show increasing variation from OK to IDW to NNP. The Ok method predominantly features red blocks (Fe >30%) across the ore body, with few yellow blocks (25% - 30% Fe) concentrated in the northwest section. The IDW method is more closely aligned with the ordinary kriging geostatistical interpolation method, displaying a similar color pattern with only a micro introduction of green blocks or low-grade blocks. In contrast, the NNP method introduces a significant number of lower-grade blocks, with a notable increase in green and blue areas throughout the ore body. This increased variability arises from the NNP method's approach of assigning the grade of the closest known sample to entire unknown blocks, resulting in a more pronounced underestimation of grades. The smoothing effect of OK method, which utilizes spatial correlation to estimate grades based on surrounding values, is evident in the more stable distribution of grades in OK compared to the more extreme values observed in NNP. OK method tends to moderate the variability of input data, producing estimates that are less variable than the original sample values (Chilès & Delfiner, 2012). The reduced variability is also reflected in the narrower range (34.79%) and lower standard deviation (4.32) compared to the inverse distance weighting method and the nearest neighbor polygon (NNP) method. The smoothing effect of ordinary kriging method enhances the stability and predictability of the model and also helps reduce the risk of overestimation or underestimation that might result from isolated high-grade making it a preferred method for global resource estimation (Bargawa & Tobing, 2020).

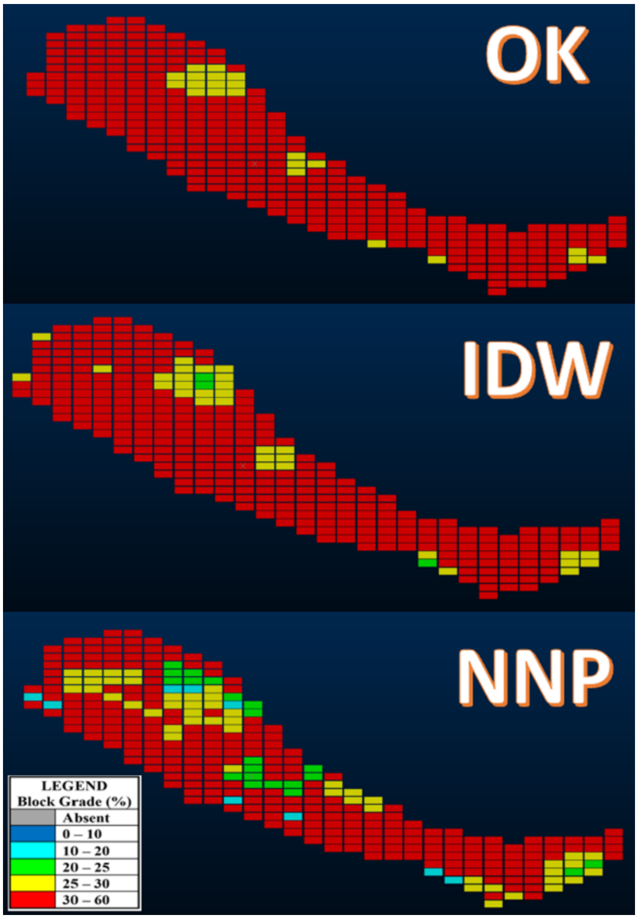

Figure 7 Comparism of estimated block model showing iron ore grades derived from Ordinary Kriging (OK), Inverse Distance Weighting (IDW), and Nearest Neighbor Polygon (NNP) interpolation method.

4.5.4. Comparison of correlation coefficient

The individual estimates of the three methods were correlated using a scatter plot and correlation coefficient value (Figure 8). Correlation coefficients illustrate the relationship between the estimation methods. The correlation coefficient of the ordinary kriging vs. inverse distance weighting method (Figure 8a) is 0.876, indicating a strong correlation between the two methods. The correlation between OK and nearest neighbor polygon (NNP) (Figure 8b) is 0.687, while the correlation between IDW and NNP (Figure 8c) is 0.720.

The high correlation coefficient of 0.876 between OK and IDW suggests that these two methods produce more comparable results. In contrast, a lower correlation is shown for the other methods. The IDW-NNP estimates, with a correlation coefficient of 0.720, show a slightly higher degree of similarity than the OK-NNP estimates, which have a correlation of 0.687, suggesting that IDW method shares a marginally greater degree of similarity with NNP than OK does.

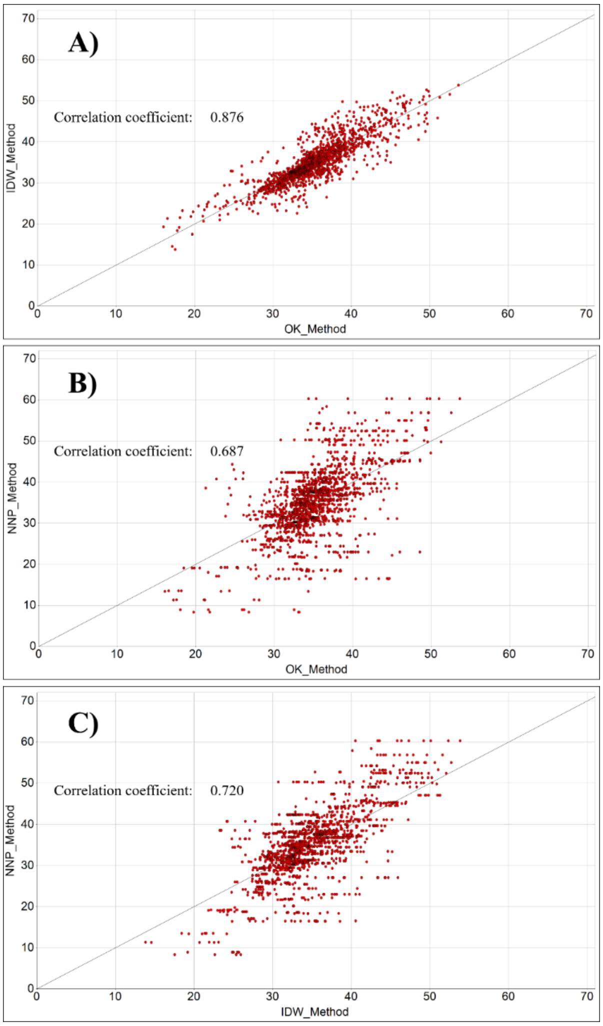

Figure 8 Scatterplot showing correlation between iron grade estimates from Ordinary Kriging (OK), Inverse Distance Weighting (IDW), and Nearest Neighbor Polygon (NNP) methods. (a) Ok versus IDW, (b) OK versus NNP, and (c) IDW versus NNP.

4.6. Validation of Resource Estimates

The slope of regression was used to evaluate the accuracy of the ordinary kriging (OK) method by comparing the actual grade with the estimated grade (Z/Z*). The goal is to have a mean slope of regression value equal to 1 or close to 1. Ideally, a mean slope of regression value equal to 1 or close to 1 indicates high estimation accuracy. As shown in Figure 9, a histogram of the calculated slope of regression values shows about 85% of the kriged estimates falling within the range of 0.5 to 1, indicating good correlation. The mean slope of regression value for these estimates is 0.792, indicating a good level of accuracy.

In addition, the three estimation methods - nearest neighbor polygon (NNP), inverse distance weighting (IDW), and ordinary kriging (OK) - were validated and compared using statistical measures: the coefficient of variation, standard error, and the global mean grade difference vs. model. Table 5 shows a standard error and the global mean grade difference vs. model %. The ordinary kriging method and inverse distance weighting method standard error is 0.105 and 0.106, respectively, indicating high accuracy. In contrast, the nearest neighbor polygon had a higher standard error of 0.206, indicating reduced accuracy. Regarding global mean grade difference relative to the model, the inverse distance weighting method had the smallest deviation followed by the ordinary kriging method, while the nearest neighbor polygon shows the largest deviation. Generally, a global mean grade vs. model difference value that is less than 5% indicates high accuracy estimation.

Table 5 Validation of estimation methods.

	Validation Method	
	Standard error	% difference Global mean grade vs model
OK	0.105	2.21%
IDW	0.106	0.96%
NNP	0.206	2.76%

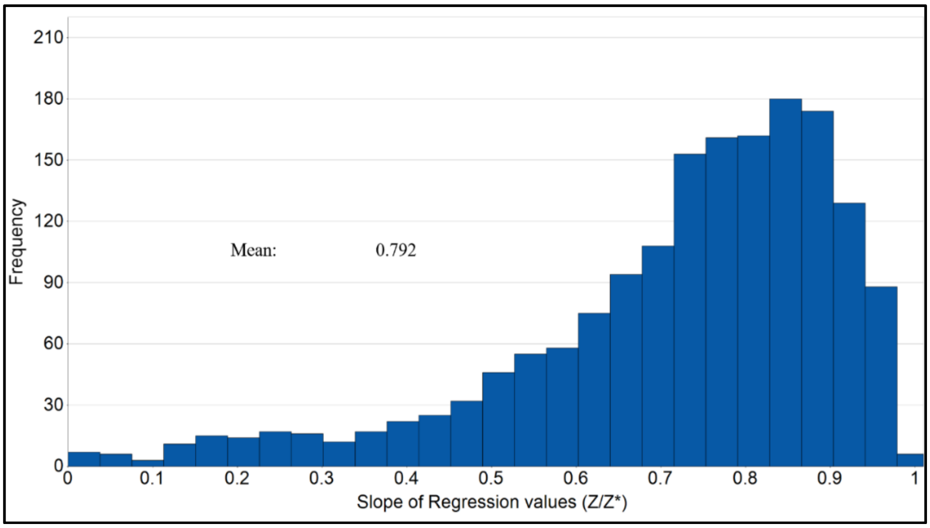

Figure 9 Histogram of slope of regression values used to validate Ordinary Kriging (OK) estimates for the Gofolo Hill deposit.

5. Conclusion

This study applied the ordinary kriging geostatistical method and the inverse distance weighting and nearest neighbor polygon traditional method to estimate the mineral resource of the Gofolo Hill iron ore deposit in Western Liberia. The ordinary kriging method estimated a total resource of 17.169 million tonnes at an average grade of 35.90%, while the inverse distance weighting method estimated 16.975 million tonnes at an average grade of 35.53%, and the nearest neighbor polygon method estimated 14.757 million tonnes at an average grade of 38.55%.

Comparative analysis of the three methods - using statistical analysis, visualization of grade blocks, resource estimates, and correlation coefficient – indicates that ordinary kriging is more precise. The inverse distance weighting method closely followed showing a strong consistency with OK (correlation coefficient: 0.876), while the nearest neighbor polygon method showed the least accuracy and precision and highest variability reflected in its broader range, higher standard deviation and coefficient of variation.

Validated by the standard error value close to zero and the percent mean grade difference vs. model below 5% confirmed the accuracy of all three methods. Additionally, the ordinary kriging method showed reliable predictions and a good correlation between estimated and actual grades, with a mean slope of regression value of 0.792.

The findings highlight the importance of selecting the appropriate interpolation method in mineral resource estimation and demonstrate the effectiveness of geostatistical techniques in accurately modeling iron ore deposits.

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